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Univerzitet Crne Gore

S e n a t u

Odboru za doktorske studije

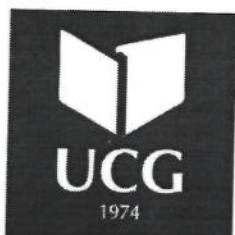
U skladu sa članom 41 Pravila doktorskih studija, u prilogu akta dostavljamo predlog Odluke Vijeća o imenovanju Komisije za ocjenu doktorske disertacije mr Milice Kankaraš pod nazivom "Reducibilnost u algebarskim hiperstrukturama" radi davanja saglasnosti.

S poštovanjem,

Za Dekan,

Prof. dr Predrag Miranović





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Na osnovu člana 64 Statuta Univerziteta Crne Gore i člana 38 stav s Pravila doktorskih studija, Vijeće Fakulteta na LXIV sjednici održanoj 06.07.2021.godine, donijelo je

O D L U K U

Predlažemo Centru za doktorske studije i Senatu Univerziteta Crne Gore da imenuje Komisiju za ocjenu doktorske disertacije mr Milice Kankaraš pod nazivom "Reducibilnost u algebarskim hiperstrukturama" u sastavu:

1. Dr Michal Noval, docent ETF-a Univerziteta Tehnologije u Brnu (Češka Republika) (naučna oblast: Algebra);
2. Dr Svetlana Terzić, redovni profesor PMF-a UCG (naučna oblast: Algebarska topologija);
3. Dr Biljana Zeković, redovni profesor PMF-a UCG, (naučna oblast: Algebra);
4. Dr Sanja Jančić- Rašović, redovni profesor PMF-a UCG (naučna oblast: Algebra) i
5. Dr Irina Elena Cristea, vanredni profesor Univerziteta Nova Gorica (Slovenija) (naučna oblast: Algebra).

Obrazloženje

Mr Milica Kankaraš podnijela je Vijeću Prirodno-matematičkog fakulteta doktorsku disertaciju pod nazivom "Reducibilnost u algebarskim hiperstrukturama". Vijeće Prirodno-matematičkog fakulteta je shodno članu 38 stav 3 Pravila doktorskih studija utvrdilo Predlog Odluke za imenovanje komisije za ocjenu doktorske disertacije.



D E K A N

Prof. dr Predrag Miranović



ISPUNJENOST USLOVA DOKTORANDA

| OPŠTI PODACI O DOKTORANDU | | | |
|--|--|--|-----------------|
| Titula, ime, ime roditelja, prezime | MSc Milica (Mitar) Kankaraš | | |
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| Broj indeksa | 1/2012 | | |
| NAZIV DOKTORSKE DISERTACIJE | | | |
| Na službenom jeziku | Reducibilnost u algebarskim hiperstrukturama | | |
| Na engleskom jeziku | Reducibility in algebraic hyperstructures | | |
| Naučna oblast | | | |
| MENTOR/MENTORI | | | |
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| Datum značajni za ocjenu doktorske disertacije | | | |
| Sjednica Senata na kojoj je data saglasnost na ocjenu temu i kandidata | 17. 5. 2019. g. | | |
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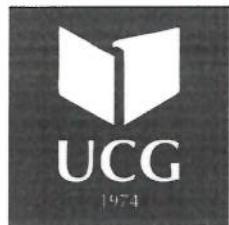
U Podgorici,
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 DEKAN

Prilog dokumenta sadrži:

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Datum: 01. 07. 2021.

Na osnovu službene evidencije i dokumentacije Prirodno-matematičkog fakulteta u Podgorici, izdaje se

P O T V R D A

MSc Milica Kankaraš, student doktorskih studija na Prirodno-matematičkom fakultetu u Podgorici, dana 01. 07. 2021. godine dostavila je ovom Fakultetu doktorsku disertaciju pod nazivom "Reducibilnost u algebarskim hiperstrukturama", na dalji postupak.

D e k a n,
dekanat
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Reducibility in Corsini hypergroups

Milica Kankaras

Abstract

In this paper, we study the reducibility property of special hypergroups, called Corsini hypergroups, named after the mathematician who introduced them. The concept of reducibility was introduced by Jantosciak, who noticed that it can happen that hyperproduct does not distinguish between a pair of elements. He defined a certain equivalences in order to identify elements which play the same rôle with respect to the hyperoperation. First we will determine specific conditions under which the Corsini hypergroups are reduced. Next, we will present some properties of these hypergroups necessary for studying the fuzzy reducibility property. The fuzzy reducibility will be considered with respect to the grade fuzzy set $\tilde{\mu}$, used for defining the fuzzy grade of a hypergroup. Finally, we will study the reducibility and the fuzzy reducibility of the direct product of Corsini hypergroups.

1 Introduction

In a classical algebraic structure (group, ring, field, etc) the result of the synthesis, called operation, between two elements of the support set is an element of the same support set. Extending this property in a "hyper" way, one can consider the synthesis of two elements having as result a subset of the support set, so substituting the operation on a set with a hyperoperation. This genial idea came to F. Marty in 1934, when he proved that the quotient structure of a group by any arbitrary subgroup can be defined as a hypergroup. This is a set H endowed with a hyperoperation, i.e. a function $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ defined from the Cartesian product $H \times H$ to the set of non-empty subsets of H , having two properties:

Key Words: Hypergroup, equivalence relations, reducibility, Corsini hypergroups.

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1. the associativity: for any $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$ and
2. the reproducibility: for any $x \in H$, $x \circ H = H = H \circ x$.

Here, $(x \circ y) \circ z$ must be read as $\bigcup_{u \in x \circ y} u \circ z$, and similarly $x \circ (y \circ z)$ must be read as $\bigcup_{u \in y \circ z} x \circ u$.

Since then, the theory of algebraic hypercompositional structures has been developed from different perspectives, becoming today not only a very well known branch of Modern Algebra, but also an important tool to solve problems in other areas, as graph theory, probability, geometry, number theory, coding theory, etc. For a collection of some applications obtained before 2003, we indicate the book [7]. One of the most studied applications of hypergroups is that one related with binary or n-ary relations ([3, 8, 9, 11, 12, 14, 16, 24]), that has been then extended to graphs and hypergraphs ([2, 17, 23, 25]). This idea was developed at the beginning by the Italian school, mainly by P. Corsini, who associated to an arbitrary hypergraph a particular commutative quasi-hypergroup and founded necessary and sufficient conditions so that it is a hypergroup [2]. Corsini called this new structure a "hypergraph-hypergroupoid". In 2019 Al Tahan and Davvaz [1] studied again this hypergroup, calling it "Corsini hypergroup", finding its properties related to cyclicity, regular relations, complete parts and direct products of hypergroups. It is also interesting to notice that a particular type of this Corsini hypergroup was studied by G. Massouros [23] for its applications in the Theory of Languages. More precisely, this is a B-hypergroup, where the hyperoperation is defined as $x \circ y = \{x, y\}$ for any two arbitrary elements. The B-hypergroup appears also in the study of fortified join hypergroups [21] or breakable semihypergroups [18].

In this paper we will study the reducibility property of Corsini hypergroups. This concept was first defined by Jantosciak [19] in 1990; when he noticed that in a hypergroup (and we can say that in any hypercompositional structure) some elements play "interchangeable roles" with respect to the hyperoperations. In particular, two arbitrary elements can belong to the same hyperproducts of elements, or their hyperproducts with all elements in the support set are the same. Mathematically speaking, two equivalence relations can be defined on a hypergroup in order to describe these properties. They have been introduced by Jantosciak [19], who called them inseparability and operational equivalence. Combining both of them, Jantosciak defined also a third equivalence, i.e. the essential indistinguishability. Moreover, he called a hypergroup to be reduced if the equivalence class of each element in the hypergroup is a singleton with respect to the last equivalence relation. This property was studied in deep by Cristea ([9, 10, 11, 14]) for hypergroups associated to binary or n-ary relations and extended also to the fuzzy case [10].

Recently, Kankaras and Cristea [20] have investigated the (fuzzy) reducibility of complete hypergroups, i.p.s hypergroups and a particular non-complete 1-hypergroups. The fuzzy reducibility was considered with respect to the grade fuzzy set $\tilde{\mu}$, introduced by Corsini [5] and studied by Corsini and Cristea for the definition of the fuzzy grade of a hypergroup [6]. For an element u in hypergroup, grade fuzzy set $\tilde{\mu}(u)$ is the average value of the reciprocals of the sizes of all hyperproducts which contain the element u . Fuzzy grade of the hypergroup represents the length of the sequence of join spaces and fuzzy sets associated with the given hypergroup.

Motivated by the above mentioned studies, in this note we aim to study the reducibility and the fuzzy reducibility property of Corsini hypergroups. First, we will recall some definitions and properties related with Corsini hypergroups, then we will establish conditions under which these hypergroups are reduced or fuzzy reduced. Finally, we will focus on the study of the product of Corsini hypergroups and its reducibility. Conclusions and new ideas for further research are covered by the last section.

2 Preliminaries

In this section we briefly recall the main definitions concerning the reducibility and the fuzzy reducibility of hypergroups, as well as the concept of Corsini hypergroup. For more details and a solid background of the theory of algebraic hypergroups the readers can consult the monographs [4, 7, 15].

Definition 2.1. [19] Two elements x, y in a hypergroup (H, \circ) are called:

1. *operationally equivalent* or by short *o-equivalent*, and write $x \sim_o y$, if $x \circ a = y \circ a$, and $a \circ x = a \circ y$, for any $a \in H$;
2. *inseparable* or by short *i-equivalent*, and write $x \sim_i y$, if, for all $a, b \in H$, $x \in a \circ b \iff y \in a \circ b$;
3. *essentially indistinguishable* or by short *e-equivalent*, and write $x \sim_e y$, if they are operationally equivalent and inseparable.

Definition 2.2. [19] A hypergroup is called *reduced* if the equivalence class of each element with respect to the essentially indistinguishable relation is a singleton.

If now we consider a fuzzy set $\mu : H \rightarrow [0, 1]$ defined on a hypergroup H , then we can extend the reducibility property to the fuzzy case. As in the classical approach, first we define three equivalences relations that keep the same model as the inseparability and operationally equivalence.

Definition 2.3. [20] In a crisp hypergroup (H, \circ) endowed with a fuzzy set μ , for two arbitrary elements $x, y \in H$, we say that:

1. x and y are *fuzzy operationally equivalent* and write $x \sim_{fo} y$ if, for any $a \in H$, $\mu(x \circ a) = \mu(y \circ a)$ and $\mu(a \circ x) = \mu(a \circ y)$;
2. x and y are *fuzzy inseparable* and write $x \sim_{fi} y$ if $\mu(x) \in \mu(a \circ b) \iff \mu(y) \in \mu(a \circ b)$, for $a, b \in H$;
3. x and y are *fuzzy essentially indistinguishable* and write $x \sim_{fe} y$, if they are fuzzy operationally equivalent and fuzzy inseparable.

Definition 2.4. [20] The crisp hypergroup (H, \circ) is a *fuzzy reduced hypergroup* if the equivalence class of each element in H with respect to the fuzzy essentially indistinguishable relation is a singleton, i.e. for all $x \in H$, $\hat{x}_{fe} = \{x\}$.

As it was already explained in [20], the fuzzy reducibility depends on the considered fuzzy set, so it can change when we consider different fuzzy sets. For any hypergroupoid (H, \circ) , the grade fuzzy set $\tilde{\mu}$ is defined as follows:

$$\tilde{\mu}(u) = \frac{A(u)}{q(u)},$$

where $A(u) = \sum_{(x,y) \in Q(u)} \frac{1}{|x \circ y|}$, $Q(u) = \{(x, y) \in H^2 : u \in x \circ y\}$, $q(u) = |Q(u)|$. For $Q(u) = \emptyset$, by default we take $\tilde{\mu}(u) = 0$.

In the first studies concerning the relationship between hypergroups and hypergraphs, Corsini defined the following hypergroupoid.

Definition 2.5. [2] On a non empty set H , define the hyperoperation \circ' as follows. For all $(x, y) \in H^2$,

1. $x \circ' y = x \circ x \cup y \circ y$,
2. $x \in x \circ x$,
3. $y \in x \circ x \iff x \in y \circ y$.

Theorem 2.6. [2] A hypergroupoid (H, \circ) satisfying the conditions in Definition 2.5 is a hypergroup if and only if also the following condition is valid:

$$\forall (a, c) \in H^2 \quad c \circ c \circ c \setminus c \circ c \subseteq a \circ a \circ a.$$

This hypergroup was studied also in [1], where the authors named it "Corsini hypergroup" and investigated also its properties connected with the Cartesian product. Here we recall one result, that we will need in our research.

Theorem 2.7. [1] Let (H, \circ_1) and (H, \circ_2) be two Corsini hypergroups. Then the direct product of hypergroups $(H \times H, \circ_1 \times \circ_2)$ is a Corsini hypergroup if and only if (H, \circ_1) or (H, \circ_2) (or both) is a total hypergroup.

Note that, for two given hypergroups defined on the same support set H , the hyperoperation $\otimes = \circ_1 \times \circ_2$ is defined as $(x_1, x_2) \otimes (y_1, y_2) = (x_1 \circ_1 y_1, x_2 \circ_2 y_2), x_1, x_2, y_1, y_2 \in H$. The structure $(H \times H, \otimes)$ is called the *direct product of hypergroups*.

We end this preliminary section with one particular type of Corsini hypergroup, studied for its important properties in the theory of automata and languages [22], and called *B-hypergroup* by G. Massouros, after the binary result that the hyperoperation gives. It was also investigated in connection with fortified join spaces [21] or breakable semihypergroups [18].

Definition 2.8. [22] Let H be any non-empty set. For any $(x, y) \in H^2$, define $*$ as follows

$$x * y = \{x, y\}.$$

Then the hypergroup $(H, *)$ is called a B-hypergroup.

Proposition 2.9. [1] Any B-hypergroup $(H, *)$ is a Corsini hypergroup.

3 The reducibility in Corsini hypergroups

In this section we determine necessary and sufficient condition for the Corsini hypergroup to be reduced. Secondly, we prove that any B-hypergroup is always reduced. Also, we give an example of a reduced hypergroup which is not a B-hypergroup.

Proposition 3.1. Let (H, \circ) be a Corsini hypergroup. If there exist some different elements x, y in H such that $x \circ x = y \circ y$, then the hypergroup (H, \circ) is not reduced.

Proof. Let x, y be arbitrary elements in H such that $x \neq y$ and $x \circ x = y \circ y$. It is easy to see that $x \circ a = y \circ a$, for any $a \in H$, since $x \circ a = x \circ x \cup a \circ a = y \circ y \cup a \circ a = y \circ a$. Using the commutativity, we obtain that $a \circ x = a \circ y$, for any $a \in H$. Hence, $x \sim_o y$. Let $\bar{x} \in c \circ d$, with $x, c, d \in H$. Then $\bar{x} \in c \circ c \cup d \circ d$, which implies that $\bar{x} \in c \circ c$ or $\bar{x} \in d \circ d$. Since (H, \circ) is a Corsini hypergroup, the previous implication gives $c \in x \circ x$ or $d \in x \circ x$ and $c \in y \circ y$ or $d \in y \circ y$. Using the same property, we conclude that $y \in c \circ d$. Similarly, one proves the converse implication. Therefore, $x \sim_i y$. Hence, the hypergroup (H, \circ) is not reduced. \square

As a consequence of Proposition 3.1, we obtain the following results.

Proposition 3.2. *A Corsini hypergroup (H, \circ) with at least two different elements is reduced if and only if $x \circ x \neq y \circ y$, for all $x, y \in H$.*

Proof. The contraposition of Proposition 3.1 directly gives the first direction. Suppose now that $x \circ x = y \circ y$, for all $x, y \in H$. Take two arbitrary elements $x \neq y$ from H . We will prove that $x \circ a = y \circ a$, for all $a \in H$, just in case when $x = y$. Assume that $x \circ a = y \circ a$, for all $a \in H$. From here, we have $x \circ x = y \circ x$, which gives $x \circ x = y \circ y \cup x \circ x$. The last equality is possible only if $y \circ y \subseteq x \circ x$. Similarly, since $x \circ y = y \circ y$, it follows the other inclusion $x \circ x \subseteq y \circ y$. Therefore, $x \circ a = y \circ a$ is equivalent with $x \circ x = y \circ y$, which contradicts the hypothesis. Hence, two arbitrary elements x and y , $x \neq y$ are not operationally equivalent, thus $\hat{x}_e = \{x\}$ for all $x \in H$, meaning that H is a reduced hypergroup. \square

Proposition 3.3. *Any B-hypergroup is reduced.*

Proof. This immediately follows from Proposition 3.2, since in a B-hypergroup there is $x \circ x = \{x\}$, for all elements x . \square

In the following example we present a reduced Corsini hypergroup, which is not a B-hypergroup.

Example 3.4. On the set $H = \{a, b, c\}$ define the hyperoperation " \circ " by the following table:

| \circ | a | b | c |
|---------|-----|--------|--------|
| a | H | H | H |
| b | H | a, b | H |
| c | H | H | a, c |

Since all the rows in the table are different, it follows that $\hat{x}_o = \{x\}$ for any $x \in H$, which clearly implies the reducibility of the hypergroup.

4 Fuzzy reducibility in Corsini hypergroups

The aim of this section is to prove that a Corsini hypergroup (H, \circ) is not fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$. For doing this, first we present some properties regarding the hyperproducts $x_i \circ x_i$, with $x_i \in H$. For a finite hypergroup H with n elements, we will denote its cardinality by $|H| = n$. Recall also that, for any $u \in H$, $\tilde{\mu}(u) = \frac{A(u)}{q(u)}$, where $A(u) = \sum_{(x,y) \in Q(u)} \frac{1}{|x \circ y|}$, $Q(u) = \{(x, y) \in H^2 : u \in x \circ y\}$, $q(u) = |Q(u)|$.

Proposition 4.1. *Let (H, \circ) be a Corsini hypergroup with n elements. If an element x_i appears in exactly k hyperproducts $x_j \circ x_j$, $j = 1, 2, \dots, n$, then $q(x_i) = 2nk - k^2$.*

Proof. Let x_i be an arbitrary element from $H = \{x_1, x_2, x_3, \dots, x_n\}$ which appears in k hyperproducts $x_j \circ x_i$, for some $j = 1, \dots, n$. By the definition of the hyperoperation of a Corsini hypergroup, it follows that x_i appears in every hyperproduct $x_j \circ x_k$, with $k \in \{1, \dots, n\}$. For one fixed k , because the commutativity, x_i appears in $n + n - 1$ hyperproducts. The sum of all such cases is:

$$(2n-1) + (2n-1) - 2 \cdot 1 + (2n-1) - 2 \cdot 2 + \dots + (2n-1) - 2 \cdot (k-1) = \\ (2n-1) \cdot k - 2(1+2+\dots+k) = (2n-1) \cdot k - (k-1)k = 2nk - k^2.$$

□

Proposition 4.2. *The sum of all cardinalities of $x_i \circ x_i, x_i \in H$ when $|H|$ is odd (even) is an odd (even) number.*

Proof. Let $|H| = n$ be an even number. If $|x_i \circ x_i| = 1$, for every $x_i \in H$, then $\sum_i^n |x_i \circ x_i| = 1 \cdot n = n$ which is an even number. Let add k elements to a hyperproduct $x_i \circ x_i, k \leq n-1$. In that case, by the property 3 of the definition of the hyperoperation " \circ ", we have to add the element x_i to k -hyperproducts $x_j \circ x_j$. All together, we add $k+k = 2k$ elements, which is again an even number. Continuing this process, so adding an arbitrary number of elements to any hyperproduct $x_i \circ x_i$, we always get an even number. Summing arbitrary even numbers, we obtain at the end an even number. The proof is analogous in the case when n is an odd number. □

Proposition 4.3. *Let (H, \circ) be a Corsini hypergroupoid of cardinality n . The number of all possible different sums of the cardinalities of the hyperproducts $x_i \circ x_i, x_i \in H$, is $\frac{n^2-n}{2} + 1$.*

Proof. The proof will be performed using the mathematical induction. For hypergroups of cardinality 2, the property is easily satisfied, because if H contains two elements, we have exactly two possibilities. The hyperproducts $x \circ x$ are singleton, or equal to H . In the first case the sum of the cardinalities of $x_i \circ x_i$ is 2, while in the second case the sum is 4. Thus, the number of the different sums is 2, i.e. $\frac{2^2-2}{2} + 1$. Assume that for $|H| = n$ the number of the different sums is equal to $\frac{n^2-n}{2} + 1$. Let us prove that the claim is valid for $|H| = n+1$. In this case we have to analyse only the hyperproduct $x_{n+1} \circ x_{n+1}$. If $x_{n+1} \circ x_{n+1} = \{x_{n+1}\}$, then we have $\frac{n^2-n}{2} + 1$ possible sums, i.e. the number of sums is the same as in the inductive case. The other cases are: $x_{n+1} \circ x_{n+1} = \{x_{n+1}, x_i\}, x_{n+1} \circ x_{n+1} = \{x_{n+1}, x_i, x_j\}, \dots, x_{n+1} \circ x_{n+1} = H$. It gives n sums more, which is finally $\frac{n^2-n}{2} + 1 + n$, i.e. number of possible sums when $|H| = n+1$ is equal to $\frac{(n+1)^2-(n+1)}{2} + 1$, which proves the proposition. □

Remark 4.4. Let (H, \circ) be a Corsini hypergroup of cardinality n . There are at least $\frac{n^2-n}{2}+1$ Corsini hypergroups of order n up to isomorphism. Since the hyperproducts $x \circ x, x \in H$ completely determine the hypergroup, it follows that $\frac{n^2-n}{2}+1$ different sums define at least as many different hypergroups. One sum can form more different tables, and in case when $n \geq 3$ the number of hypergroups is greater.

Proposition 4.5. Let (H, \circ) be a Corsini hypergroup of cardinality n . If an element x_i appears in k hyperproducts $x_j \circ x_j$, and if we assume that the cardinalities of those sets are, respectively m_1, m_2, \dots, m_k , then

$$\tilde{\mu}(x_i) = \frac{\frac{1}{m_1} + \frac{1}{m_2} + \dots + \frac{1}{m_k} + 2 \cdot \sum_{\substack{i \neq j \\ i=1, \dots, k}} \frac{1}{|x_i \circ x_j|}}{2nk - k^2}$$

Proof. According to definition of the fuzzy grade set $\tilde{\mu}$ and Proposition 4.1, the result is clearly satisfied. \square

Remark 4.6. If two elements of a Corsini hypergroup have the same number of appearances in some hyperproducts $x_j \circ x_j$, and the cardinalities of those hyperproducts are the same for both elements, based on Proposition 4.5, then their values under the grade fuzzy set $\tilde{\mu}$ are the same. Hereinafter, we will say that elements with this property are in the *same formation*.

Proposition 4.7. In any Corsini hypergroup (H, \circ) , the fuzzy operational equivalence implies the fuzzy inseparability.

Proof. Let $x, y \in H$ be two arbitrary elements in H such that $x \sim_{fo} y$, i.e. $\tilde{\mu}(x \circ a) = \tilde{\mu}(y \circ a)$, for $\forall a \in H$. It means that:

$$\begin{aligned} & \tilde{\mu}(x \circ x \cup a \circ a) = \tilde{\mu}(y \circ y \cup a \circ a) \\ \iff & \tilde{\mu}(x \circ x) \cup \tilde{\mu}(a \circ a) = \tilde{\mu}(y \circ y) \cup \tilde{\mu}(a \circ a). \end{aligned}$$

Since this equality is satisfied for every set $\tilde{\mu}(a \circ a), a \in H$, it follows that $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$ and contains both $\tilde{\mu}(x)$ and $\tilde{\mu}(y)$ by property 3. of Definition 2.5. If $\tilde{\mu}(x) = \tilde{\mu}(y)$, then clearly $x \sim_{fi} y$. Let us consider now the case when $\tilde{\mu}(x) \neq \tilde{\mu}(y)$. Suppose that $\tilde{\mu}(x) \in \tilde{\mu}(c \circ d) = \tilde{\mu}(c \circ c) \cup \tilde{\mu}(d \circ d)$. Let us take $\tilde{\mu}(x) \in \tilde{\mu}(c \circ c)$. It follows that, for some $z \in c \circ c$, $\tilde{\mu}(x) = \tilde{\mu}(z)$. The equality $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$ means that $\{\tilde{\mu}(l) \mid l \in x \circ x\} = \{\tilde{\mu}(k) \mid k \in y \circ y\}$, i.e. for every $l \in x \circ x$ there exists $k \in y \circ y$ such that $\tilde{\mu}(l) = \tilde{\mu}(k)$. Now, since $\tilde{\mu}(x) = \tilde{\mu}(z) \in \tilde{\mu}(x \circ x)$, $\tilde{\mu}(x) \in \tilde{\mu}(x \circ x)$, and $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$, we

conclude that $\tilde{\mu}(z) \in \tilde{\mu}(y \circ y)$. Thus there exists $l \in y \circ y$ such that $\tilde{\mu}(z) = \tilde{\mu}(l)$. But $\tilde{\mu}(z) \in \tilde{\mu}(c \circ c)$, so $\tilde{\mu}(l) \in \tilde{\mu}(c \circ c)$, with $l \in y \circ y$, which finally gives $\tilde{\mu}(y) \in \tilde{\mu}(c \circ c)$. The converse implication can be proved taking $\tilde{\mu}(y) \in \tilde{\mu}(c \circ c)$ and proving that $\tilde{\mu}(x) \in \tilde{\mu}(c \circ c)$. This shows that $\tilde{\mu}(x)$ and $\tilde{\mu}(y)$ appear in the same $\tilde{\mu}(c \circ c)$. Finally, according to the definition of $\tilde{\mu}(x \circ y)$, it is easy to prove that the previous equivalence implies the fuzzy inseparability. \square

Proposition 4.8. *Let (H, \circ) be a Corsini hypergroup of cardinality n . If x is an element such that $x \circ x$ is a singleton, i.e. $x \circ x = \{x\}$, then $\tilde{\mu}(x) = \frac{1+2 \cdot \sum_{a \neq x} \frac{1}{|x \circ a|}}{2n-1}$, with $a \in H$.*

Proof. Using Proposition 4.1 we easily get that $q(x) = 2n - 1$. Since x appears in every product $x \circ a, a \in H$, and the commutativity holds, then $A(x) = 1 + 2 \cdot \sum_{a \neq x} \frac{1}{|x \circ a|}$, which clearly gives the formula. \square

Based on this result, we can state sufficient conditions such that two elements in a Corsini hypergroup are fuzzy essentially indistinguishable.

Proposition 4.9. *If there exist two elements x, y in a Corsini hypergroup (H, \circ) such that $x \circ x = x$ and $y \circ y = y$, then $x \sim_{fe} y$.*

Proof. Using Proposition 4.8 this obviously holds, because $\tilde{\mu}(x) = \tilde{\mu}(y)$. \square

Proposition 4.10. *If there exist two elements x, y in Corsini hypergroup (H, \circ) such that $x \circ x = y \circ y = H$, then $x \sim_{fe} y$.*

Proof. Since $x \circ x = H$, based on condition 3 of Definition 2.5 it follows that x appears in all hyperproducts $z \circ z$, with $z \in H$, and similarly holds for y . So x and y are in the same formation. According to Proposition 4.5, we have $\tilde{\mu}(x) = \tilde{\mu}(y)$, so x and y are fuzzy inseparable. Besides, $\mu(x \circ a) = \mu(y \circ a) = \mu(\{x \mid x \in H\})$, which implies the fuzzy operational equivalence. Therefore, $x \sim_{fe} y$. \square

Theorem 4.11. *Any B-hypergroup is not fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.*

Proof. Regarding to the definition of a B-hypergroup, we have $|x \circ x| = 1$ and $|x \circ a| = 2$ for every $x \neq a$, so $A(x) = 1 + 2 \cdot (n - 1) \cdot \frac{1}{2} = n$. Using Proposition 4.1, we know that $q(x) = 2n - 1$, which clearly gives that, for any $x \in H$, $\tilde{\mu}(x) = \frac{n}{2n-1}$. Hence, two arbitrary elements in a B-hypergroup are fuzzy inseparable. Besides, $\tilde{\mu}(x \circ a) = \tilde{\mu}(y \circ a)$, for any $a \in H$ since $\tilde{\mu}(x) = \tilde{\mu}(y)$ for two arbitrary elements from H , and $\tilde{\mu}(x \circ a) = \tilde{\mu}(\{x, a\}) = \{\tilde{\mu}(x), \tilde{\mu}(a)\}$. \square

Proposition 4.12. *Let (H, \circ) be a Corsini hypergroup with $|H| \geq 2$. There always exist two elements $x, y \in H$ such that $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$.*

Proof. We will split the proof in some cases. Using Propositions 4.9 and 4.10 we can eliminate the cases when there exist $x, y \in H$ such that $x \circ x$ and $y \circ y$ are singleton or equal to H . It remains then to consider other three cases.

1. There exists $x \in H$ such that $x \circ x = H$.
2. There exists $x \in H$ such that $x \circ x = x$,
3. The hypergroup doesn't contain any element x such that $x \circ x$ is equal to x or H .

Case 1. Without losing the generality, assume that $H = \{x_1, x_2, \dots, x_n\}$ and $x_n \circ x_n = H$. This means that any $x_i \in H$ belongs to $x_n \circ x_n$, that implies $x_n \in x_i \circ x_i$, for any $i = 1, 2, \dots, n$.

Subcase 1.1. If $x_i \circ x_i = \{x_i, x_n\}$, $i = 1, 2, \dots, n-1$ and $x_n \circ x_n = H$, then by Proposition 4.5, we know that $\tilde{\mu}(x_i)$ is the same, for all $i = 1, 2, \dots, n-1$. This also implies that $\tilde{\mu}(x_1 \circ x_1) = \tilde{\mu}(x_2 \circ x_2) = \dots = \tilde{\mu}(x_{n-1} \circ x_{n-1})$, which concludes the result.

Subcase 1.2. Extending the previous subcase, that can be considered as a "base case", we can analyze now the situation when we add another element x_k , $k \neq n \neq i$, to the hyperproduct $x_i \circ x_i$. This leads to have $x_k \circ x_k = x_i \circ x_i = \{x_k, x_i, x_n\}$, which clearly gives $\tilde{\mu}(x_i \circ x_i) = \tilde{\mu}(x_k \circ x_k)$, which proves the proposition. Continuing the process, we can extend now this subcase into two ways:

- by adding another element to a hyperproduct $x \circ x$, with $x \in H \setminus \{x_i, x_k, x_n\}$ and again we obtain the conclusion of the result, or
- by adding a different element x_l to one of the hyperproducts $x_i \circ x_i$ or $x_k \circ x_k$. Suppose that we add it to $x_i \circ x_i$. Thus we get $x_i \circ x_i = \{x_i, x_k, x_l, x_n\}$, $x_l \circ x_l = \{x_l, x_i, x_n\}$, $x_k \circ x_k = \{x_k, x_i, x_n\}$, meaning that x_l and x_k are in the same formations, so $\tilde{\mu}(x_k) = \tilde{\mu}(x_l)$ and thereby $\tilde{\mu}(x_k \circ x_k) = \tilde{\mu}(x_l \circ x_l)$.

Continuing this process by the above described procedure, we will always get two distinct elements such that $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$. The process is finite, since we stop when we get two hyperproducts $x \circ x = H$.

Case 2. There exists $x_i \in H$ such that $x_i \circ x_i = x_i$. First, the "base case" is when all the other hyperproducts $x \circ x$, with $x \in H \setminus \{x_i\}$, contain two elements. This is possible only if the cardinality of H is odd. If the cardinality of H is an even number, the "base case" is when one hyperproduct $x_j \circ x_j$,

with $j \neq i$, has three elements, and all the other hyperproducts $x \circ x$ have exactly two elements. The value $\tilde{\mu}(x_i)$ of all elements x_i such that $|x_i \circ x_i| = 2$ is the same. Repeating the same procedure as in Case 1, we will always obtain two elements x and y which satisfy the result.

Case 3. There doesn't exist x_i such that $x_i \circ x_i = H$ nor $x_i \circ x_i = x_i$. The "base cases" are exactly the same as in the second case and they depend on the parity of the cardinality of H . For example, in the case when cardinality is an even number, we can set hyperproducts as: $x_1 \circ x_1 = x_2 \circ x_2 = \{x_1, x_2\}$, $x_3 \circ x_3 = x_4 \circ x_4 = \{x_3, x_4\}, \dots, x_{n-1} \circ x_{n-1} = x_n \circ x_n = \{x_{n-1}, x_n\}$. The values of all $\tilde{\mu}(x_i)$ are the same for all $i \in \{1, 2, \dots, n\}$, so $\tilde{\mu}(x_i \circ x_i)$ are also the same for $i \in \{1, 2, \dots, n\}$. In the case when the cardinality is an odd number, we can form hyperproducts $x_i \circ x_i$ as in the previous case for $i = 2, \dots, n-1$, but take $x_1 \circ x_1 = \{x_1, x_2, x_n\}$, $x_n \circ x_n = \{x_n, x_1\}$. This case reduces to the first case, too. Using already mentioned procedure of constructing other Corsini hypergroups, we will always get two elements x, y such that $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$. \square

It is worth noticing that the procedure described above permits us to construct all finite Corsini hypergroups.

Theorem 4.13. *Any Corsini hypergroup is not fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.*

Proof. According to Proposition 4.12 we can always find two elements x and y such that $\tilde{\mu}(x \circ x) = \tilde{\mu}(y \circ y)$. This implies the fuzzy operational equivalence of these two elements. From here, according to Proposition 4.7, we conclude that they are also fuzzy inseparable. Hence, in any Corsini hypergroup there always exist two elements in the same equivalence class with respect to the fuzzy essential indistinguishability, which gives that the hypergroup is not fuzzy reduced, with respect to the grade fuzzy set $\tilde{\mu}$. \square

Remark 4.14. Do to a manner of construction of Corsini hypergroups, showed in the Proposition 4.12, it is easy to conclude that the infinite Corsini hypergroup is also not fuzzy reduced with respect to the $\tilde{\mu}$.

Example 4.15. On the set $H = \{1, 2, 3, \dots, n\}$ let define the hyperoperation \circ_ρ by $x \circ_\rho y = x \circ_\rho x \cup y \circ_\rho y$, where $x \circ_\rho x = \{z \mid x \rho z\}$ and the relation ρ is defined as $x \rho y \iff x \leq y$. Then (H, \circ_ρ) is fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.

Indeed, note that $i \circ n = \{1, 2, 3, \dots, \max\{i, n\}\}$. Since 1 is the smallest element in the set H , then $1 \circ i = i \circ 1 = \{1, 2, \dots, i\}$, for any $i \in H$. Here, 1 appears in any hyperproduct, so $q(1) = n^2$, and the cardinalities of the sets where 1 appears are: $1, 2, \dots, n$, respectively. Similarly, $2 \circ i = i \circ 2 = \{1, 2, 3, \dots, i\}$,

and $q(2) = n^2 - 1$, because 2 doesn't appear only in the hyperproduct $1 \circ 1$. The element 2 appears in the sets of cardinalities $2, 3, 4, \dots, n-1$ respectively. For an arbitrary element k , we can conclude that it doesn't appear in hyperproducts $j \circ i$ and $i \circ j$ where $i, j \leq k$. Cardinalities of the sets where k appears are $k, k+1, \dots, n$, because k appears in every $i \circ j$, where i or j are greater than or equal to k . The set of cardinality n where k appears is every set $i \circ n$, for any $i \leq n$. Using the commutativity we conclude that we have a $2n-1$ such sets. Similarly, the set of cardinality $n-1$ where k appears is every set $i \circ (n-1)$, $i \leq n-1$ and the number of them is $2(n-1)+1$. Continuing the procedure, we get that the set of cardinality k where k appears is $i \circ k$, $i \leq k$ and the number of them is $(2k-1)$. Calculating $A(k)$, we get that k appears in $(2k-1) + (2(k+1)-1) + \dots + (2n-1)$ hyperproducts, which finally gives:

$$\tilde{\mu}(k) = \frac{\frac{1}{k} \cdot (k+k-1) + \frac{1}{k+1} \cdot (k+1+k-1-1) + \dots + \frac{1}{n} \cdot (n+n-1)}{(2k-1) + (2k+1) + (2k+3) + \dots + (2n-1)}.$$

By summing and arranging members we get $\tilde{\mu}(k) = \frac{2(n-k+1) - (\frac{1}{k} + \frac{1}{k+1} + \dots + \frac{1}{n})}{(n+k-1)(n-k+1)}$. By simple calculations it can be proved that $\tilde{\mu}(k+1) \leq \tilde{\mu}(k)$, hence k and $k+1$ are not fuzzy essentially-indistinguishable. From the previous inequality we have $\tilde{\mu}(1) \geq \tilde{\mu}(2) \geq \dots \geq \tilde{\mu}(n)$ so the equivalence class of any element in H is a singleton. Hence, (H, \circ) is fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.

Remark 4.16. Notice that the previous hypergroup is not a Corsini one, but it satisfies the first two conditions of Definition 2.5.

5 Reducibility of the direct product of Corsini hypergroups

We start this section by stating one known result about the reducibility of the product of hypergroups. After that, we study the fuzzy reducibility of the product of two non-fuzzy reduced hypergroups, that will be used for the examination of the fuzzy reducibility of the direct product of Corsini hypergroups.

Theorem 5.1 ([12]). *The hypergroup $(H \times H, \otimes)$ is reduced if and only if the hypergroups (H, \circ_1) and (H, \circ_2) are reduced.*

Proposition 5.2 ([13]). *If $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are the grade fuzzy sets of H_1 and H_2 , and $\tilde{\mu}$ is the grade fuzzy set of the direct product $H_1 \times H_2$ then $\tilde{\mu}(x, y) = \tilde{\mu}_1(x) \cdot \tilde{\mu}_2(y)$, $x, y \in H$.*

Proposition 5.3. *Let (H, \circ_1) and (H, \circ_2) be non-fuzzy reduced hypergroups constructed on the support set H with at least two elements. Then the direct product $(H \times H, \circ_1 \times \circ_2)$ is a non-fuzzy reduced hypergroup with respect to the grade fuzzy set $\tilde{\mu}$.*

Proof. For two elements a and b , we know that $\mu(a \circ b) = \{\mu(x) \mid x \in a \circ b\}$. Since (H, \circ_1) is not fuzzy reduced, assume that x_1, x_2 are two elements such that $x_1 \sim_{f.e.} x_2$, i.e. $\tilde{\mu}_1(x_1 \circ_1 a) = \tilde{\mu}_1(x_2 \circ_1 a)$, for all $a \in H$. Also, $\tilde{\mu}_1(x_1)$ and $\tilde{\mu}_1(x_2)$ appear in the same $\tilde{\mu}_1(a \circ b)$, $a, b \in H$. Similarly, since (H, \circ_2) is not fuzzy reduced, let y_1 and y_2 be elements in H such that they are fuzzy essential indistinguishable. Our goal is to prove that the ordered pairs (x_1, y_1) and (x_2, y_2) are fuzzy essential indistinguishable. Since $(x_1, y_1) \circ_1 \times \circ_2 (a, b) = (x_1 \circ_1 a, y_1 \circ_2 b)$, it follows that $\tilde{\mu}((x_1, y_1) \circ_1 \times \circ_2 (a, b)) = \{\tilde{\mu}_1(x) \cdot \tilde{\mu}_2(y) \mid x \in x_1 \circ_1 a, y \in y_1 \circ_2 b\}$. Denote the last set with A and the set $\mu((x_2, y_2) \circ_1 \times \circ_2 (a, b))$ with B . Since $x_1 \sim_{f.e.} x_2$, we have $\{\tilde{\mu}_1(x) \mid x \in x_1 \circ_1 a\} = \{\tilde{\mu}_1(y) \mid y \in x_2 \circ_1 a\}$, and $y_1 \sim_{f.e.} y_2$ implies $\{\tilde{\mu}_2(x) \mid x \in y_1 \circ_2 b\} = \{\tilde{\mu}_2(y) \mid y \in y_2 \circ_2 b\}$, meaning that $A = B$. This proves the fuzzy operational equivalence of the corresponding elements. For the proof of the fuzzy inseparability, let a, c be elements from H such that $\tilde{\mu}_1(x_1) \in \tilde{\mu}_1(a \circ_1 c)$. From here, due to the fuzzy inseparability in (H, \circ_1) , $\tilde{\mu}_1(x_2)$ belongs to the same set. On the other side, let b, d be elements from H such that $\tilde{\mu}_2(y_1) \in \tilde{\mu}_2(b \circ_2 d)$, from where we conclude that $\tilde{\mu}_2(y_2) \in \tilde{\mu}(b \circ_2 d)$. Using the last two implications, we get:

$$\begin{aligned}\tilde{\mu}_1(x_1) \cdot \tilde{\mu}_2(y_1) &\in \{\tilde{\mu}_1(x) \cdot \tilde{\mu}_2(y) : x \in a \circ_1 c, y \in b \circ_2 d\} = \\ &\{ \tilde{\mu}(x, y) : x \in a \circ_1 c, y \in b \circ_2 d \} = \tilde{\mu}(a \circ_1 c, b \circ_2 d)\end{aligned}$$

This means that $\tilde{\mu}(x_1, y_1) \in \tilde{\mu}(a \circ_1 c, b \circ_2 d)$. The above mentioned implications show that $\tilde{\mu}(x_2, y_2)$ belongs to the same set. Similarly, one proves the converse implication. Hence, (x_1, y_1) and (x_2, y_2) are fuzzy inseparable and therefore, (H, \circ_1) and (H, \circ_2) are not fuzzy reduced. \square

Proposition 5.4. *The direct product of B-hypergroups is reduced.*

Proof. Since any B-hypergroup is reduced, this is a direct corollary of Theorem 5.1. \square

The converse of Proposition 5.3 doesn't hold, as we can see in Examples 5.5 and 5.6.

Example 5.5. Let (H, \circ_1) and (H, \circ_2) be hypergroups, where the hyperoperations " \circ_1 " and " \circ_2 " are defined by the following tables.

| \circ_1 | a | b | c | d |
|-----------|-----------|-----------|-----------|-----------|
| a | a | a | a, b, c | a, b, d |
| b | a | a | a, b, c | a, b, d |
| c | a, b, c | a, b, c | a, b, c | c, d |
| d | a, b, d | a, b, d | c, d | a, b, d |

| \circ_2 | a | b | c | d |
|-----------|-----------|-----------|-----------|-----------|
| a | b | b | a, b, c | a, b, d |
| b | b | b | a, b, c | a, b, d |
| c | a, b, c | a, b, c | a, b, c | c, d |
| d | a, b, d | a, b, d | c, d | a, b, d |

Here, we will consider fuzzy reducibility with respect to the grade fuzzy set $\tilde{\mu}$.

By easy calculations, we get: $\tilde{\mu}_1(a) = \frac{11}{21}, \tilde{\mu}_1(b) = \frac{1}{3}, \tilde{\mu}_1(c) = \frac{8}{21}, \tilde{\mu}_1(d) = \frac{8}{21}$. We can notice that the only rows which are the same are those corresponding to a and b . This implies $a \sim_{f.o} b$, which easily gives $a \sim_{f.e} b$, but here, $\tilde{\mu}(a)$ belongs to $\tilde{\mu}(a \circ a)$, while $\tilde{\mu}(b)$ does not belong to it, so $a \not\sim_{f.e} b$. Hence, $a \not\sim_{f.e} b$. It is easy to see that except a and b all other pairs of elements are not fuzzy operational equivalent, which, together with $a \not\sim_{f.e} b$ implies that $\hat{x}_{f.e} = \{x\}$, for all $x \in H$. Hence, (H, \circ_1) is fuzzy reduced.

Regarding (H, \circ_2) , due to the isomorphism of hypergroups, we get the same values of the elements under the fuzzy grade $\tilde{\mu}_2$. At the same way as for the previous hypergroup, we can conclude that (H, \circ_2) is fuzzy reduced.

Here, $(a, a) \sim_{f.o} (b, b)$, because $\tilde{\mu}((a, a) \circ_1 \times \circ_2(m, n)) = \{\tilde{\mu}_1(x) \cdot \tilde{\mu}_2(y) \mid x \in a \circ_1 m, y \in a \circ_2 n\} = \{\tilde{\mu}_1(x) \cdot \tilde{\mu}_2(y) \mid \tilde{\mu}_1(x) \in \{\frac{11}{21}, \frac{1}{3}, \frac{8}{21}\}, \tilde{\mu}_2(y) \in \{\frac{1}{3}, \frac{11}{21}, \frac{8}{21}\}\}$, where $m, n \in \{a, b, c, d\}$. This set is equal to $\tilde{\mu}((b, b) \circ_1 \times \circ_2(m, n))$.

Further more, $\tilde{\mu}(a, a) = \tilde{\mu}_1(a) \cdot \tilde{\mu}_2(a) = \frac{11}{21} \cdot \frac{1}{3} = \tilde{\mu}(b, b)$, which ensures that $(a, a) \sim_{f.e} (b, b)$. Hence, we got non-fuzzy reduced hypergroup as a direct product of two fuzzy reduced hypergroups.

Example 5.6. Let (H, \circ_1) and (H, \circ_2) be hypergroups, where the hyperoperations " \circ_1 " and " \circ_2 " are defined by the following tables:

| \circ_1 | a | b | c |
|-----------|--------|--------|-----|
| a | a, b | a, b | H |
| b | a, b | a, b | H |
| c | H | H | c |

| \circ_2 | a | b | c |
|-----------|-----|-----|-----|
| a | a | a | H |
| b | a | a | H |
| c | H | H | H |

Easy calculations of the fuzzy grade sets $\tilde{\mu}_1$ and $\tilde{\mu}_2$ show that the first hypergroup (H, \circ_1) is not fuzzy reduced, while (H, \circ_2) is fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$. As in the previous example, it can be shown that $(b, a) \sim_{f.e} (a, a)$, which proves the non-fuzzy reducibility of $(H \times H, \circ_1 \times \circ_2)$.

Proposition 5.7. *The direct product of two Corsini hypergroups is non-fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.*

Proof. Since an arbitrary Corsini hypergroup is not fuzzy reduced according to Theorem 4.13, using Proposition 5.3 it follows that the direct product of two Corsini hypergroups is not fuzzy reduced. \square

Corollary 5.8. *The direct product of a Corsini hypergroup and a total hypergroup is non-fuzzy reduced with respect to the grade fuzzy set $\tilde{\mu}$.*

Proof. This is a direct consequence of Theorem 2.7. \square

6 Conclusions and Open Problems

In this paper, we have investigated different types of Corsini hypergroups with the aim to study their reducibility and fuzzy reducibility with respect to the grade fuzzy set $\tilde{\mu}$. In the second part of the paper, we have presented some conditions which give the reducibility and fuzzy reducibility of the direct product of hypergroups of Corsini hypergroups. In a future work we will extend our study to the reducibility and fuzzy reducibility of the direct product of arbitrary hypergroups. Besides, it would be interesting to construct hyperrings composed of Corsini's hypergroups and study their reducibility.

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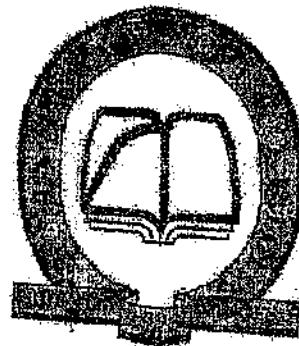
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Biografy

Milica Kankaraš was born in Nikšić on 4 April 1988. In 2006, she graduated from the high school "Stojan Cerović" in Nikšić and enrolled in the Faculty of Mathematics and Natural Sciences at the University of Montenegro, in Podgorica. In 2009, she obtained B.Sc. degree in Mathematics and Natural Sciences. One year after, she obtained a Spec. Sci degree in Mathematics and Natural Sciences. In 2012, she became a master of sciences and recently after that she started with her Phd studies. She published the article "Fuzzy reduced hypergroups" in the journal "Mathematics" together with Irina Cristea in February, 2020. In March, 2021, she published the article "Reducibility in Corsini hypergroup" in the journal "Analele Stiintifice ale Universitatii Ovidius Constanta".



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Številka: 55-3/2017

Na osnovi odločitve Senata Univerze v Novi Gorici, ki jo je v skladu z 47. in 74. členom statuta sprejel na svoji 79. seji dne 20. septembra 2017, rektor v okviru svojih pristojnosti, določenih v statutu in 20. členu Pravil o pogojih in postopku za pridobitev nazivov raziskovalnih sodelavcev, visokošolskih učiteljev in sodelavcev Univerze v Novi Gorici, sprejme naslednji ugotovitveni.

SKLEP

Doc. dr. Irini Elena Cristea se podeli naziv izredna profesorica za področje "Matematika".

Naziv velja z dnem izdaje tega sklepa za obdobje petih let.

V Novi Gorici, 22. 9. 2017

Rektor:

D. Zavrtanik

Prof. dr. Danilo Zavrtanik

Prejmejo:

- prof. dr. Irina Elena Cristea,
- personalna mapa,
- rektor,
- dekan Fakultete za naravoslovje,
- arhiv.

Name and surname: Irina Elena Cristea

Date and place of birth: 05.04.1978, Bacau (Romania)

Address: street Cormor, 48/a, 33010 Tavagnacco (Udine), Italy

Education:

- PhD in Mathematics, University of Constanta, Romania, 2007
- M.Sc. in Algebra and Geometry, University "Al.I.Cuza" Iasi, Romania, 2003
- B.Sc. in Mathematics, University "Al.I.Cuza" Iasi, Romania, 2001

Scientific title: associate professor (since 22.9.2017)

Work experience:

- 2017-..., University of Nova Gorica, School of Science, School of Environmental Sciences and School of Engineering and Management and Center for Information Technologies and Applied Mathematics- associate professor
- 2012-2017 University of Nova Gorica, School of Science, School of Environmental Sciences and School of Engineering and Management- assistant professor of mathematics
- 2009-2011, University of Udine, Italy, Faculty of mechanics - lecturer of the course "Linear Algebra" and within Department of biology and agro-industrial economy: development of computational and mathematical methods from Cluster Analysis theory for some Agro-industrial Economical problems.
- 2005-2011, University of Udine, Department of Mathematics and Informatics: development of algebraic hyperstructures theory and their applications and within Faculty of Engineering- assistant for the subjects "Linear algebra", "Calculus 1" and "Calculus 2".
- 2003-2007, University of Iasi, Romania, Faculty of mathematics- Teaching assistant

Management positions:

- Deputy head of the Center for Information Technologies and Applied Mathematics, 10.2017- present
- Deputy head of the Center for System and Information Technologies, 03.2017-10.2017, 03.2015-03.2016.

Awards: "Young scientist 2008 best paper prize" for Mathematics, awarded by University of Udine, Italy, for the paper "On the fuzzy grade of Hypergroups", published in Fuzzy Sets of Systems, 159(2008).

Projects and grants:

- Principal investigator of the Bilateral project Slovenia-Montenegro (ARRS-MS-BI-ME-JR) 2018-2020
- Principal investigator of the regional project "Borsa regionale-settore agricolo, agro-alimentare e veterinario Friuli Venezia Giulia, L.R. 2/2006 commi 54-57", University of Udine, 01.2009-09.2011
- Principal investigator of the Italian project (assegno di ricerca) "Hyperstructures, Fuzzy Sets, Rough Sets", University of Udine, Italy, 04.2007-11.2008
- Pedagogical mentor of the project "Po kreativni poti do znanja-After a creative way to knowledge", University of Nova Gorica, 03.2017-06.2017.

- Coordinator and pedagogical mentor of the project "Po kreativni poti do praktičnega znanja - After a creative way to practical knowledge", University of Nova Gorica, 04.2014-08.2014.
- Member of the project "European regional funds-Creative Cores" Active and Healthy Aging - Molecular Mechanisms, Nutrition and Targeted Delivery with Nanoparticles (AHA-MOMENT), University of Nova Gorica, 03.2015-05.2015.
- Member of the project 23/3.09.2007 of Romanian Accademy "Hipergroups and Abelian groups. Applications", University of Iasi, Romania, 09.2007-12.2007, 05.2008-10.2008.
- Member of the project CEX 05D11-11/2005 of the Romanian Ministry of Education and Science, "Combinatorial, algebraic, topological methods in Algebra and Geometry", University of Constanta, Romania, 2005-2008.

Management positions:

Head of the Center of Information Technologies and Applied Mathematics, University of Nova Gorica, 03.2017-03.2018, 03.2015-03.2016

Supervisions:

- B.Sc. Theses (University of Nova Gorica): Karolina Koren (2014), Tanja Abram (2014)
- M.Sc. Theses (University of Nova Gorica): Tamara Hohannisyan (2015)

Expert work:

- 2007- to date reviewer for more than 40 SCI mathematical journals and Mathematical Reviews

- member of the editorial board of the international journals:

1. The Scientific World Journal ISSN 1537-744X, 2013-2016
2. Italian journal of pure and applied mathematics ISSN 2239-0227, 2010-...
3. Ratio matematica ISSN 1592-7415, 2013-...
4. Population dynamics: analysis, modelling, forecast ISSN 2335-2566, 2012-...
5. Journal of advances in applied & computational mathematics ISSN 2409-5761, 2014-...
6. Journal of Hyperstructures ISSN 2251-8436, 2012-...

- International evaluator of the PhD thesis "On the study of automata and languages based on fuzzy sets, fuzzy multisets and rough sets" by Binod Kumar Sharma, submitted the Indian Institute of Technology, Dhanbad, India

Membership in scientific organizations:

- co-chair of the symposium "Hypercompositional Algebra – new Developments and Applications (1st HAnDA) within 15th International Conference on Numerical Analysis and Applied Mathematics (ICNAAM 2017)", Thessaloniki, Greece, 25-30.09.2017
- member of the organizing committee of the conferences/workshops:

1. 11th International Congress on Algebraic Hyperstructures and Applications (AHA 2011), Chieti, Italy, 17-21.10. 2011
2. A new approach in theoretical and applied methods in algebra and analysis, Constanta, Romania, 4–6.04.2013.
- member of the technical committee of the conferences:
1. Conference on Computer Science & Computational Mathematics (CCSCM 2012), Melaka, Malaysia, 9-10.02.2012.
 2. International Conference on Computer Science & Computational Mathematics (ICCSCM 2013), Kuala Lumpur, Malaysia, 9-10.02.2013
 3. International Conference on Computer Science & Computational Mathematics (ICCSCM 2014), Langkawi, Malaysia, 8-9.05.2014
 4. 12th International Congress on Algebraic Hyperstructures and Applications (AHA 2014), Xanthi, Greece, 2-7.09.2014
 5. International Conference on Computer Science & Computational Mathematics (ICCSCM 2015), Langkawi, Malaysia, 7-8.05.2015
 6. International Conference on Computer Science & Computational Mathematics (ICCSCM 2016), Langkawi, Malaysia, 5-6.05.2016
 7. International Conference on Computer Science & Computational Mathematics (ICCSCM 2017), Langkawi, Malaysia, 4-5.05.2017
 8. 13th International Congress on Algebraic Hyperstructures and Applications (AHA 2017), Istanbul, Turkey, 24-27.07.2017

Cooperation with international institutions and groups:

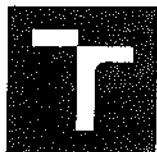
Italy (University of Udine), Romania (University "Ovidius" of Constanta, University of Bucharest), Iran (University of Tehran, University of Yazd, University of Kerman, University of Rafsanjan, Babol University of Technology, University of Bojnord, Shahid Bahonar University); Czech Republic (University of Defence of Brno, Brno University of Technology), China (Hubei Institute of Nationalities), Montenegro (University of Montenegro), Saudi Arabia (Majmaah University)

Research interest: theory of algebraic hyperstructures and their connections with fuzzy sets

Papers: 39 articles published in WoS and 45 records published in Scopus, h-index=12

Scientific monography:

B. Davvaz, I. Cristea, Fuzzy algebraic hyperstructures: an introduction, (Studies in fuzziness and soft computing, vol. 321), Springer, 2015.



REKTOR

Vážený pan

RNDr. MICHAL NOVÁK, Ph.D.

nar. 4. července 1975

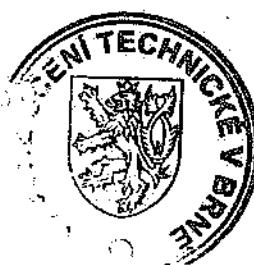
Na návrh Vědecké rady Fakulty strojního inženýrství Vysokého učení
technického v Brně, podle § 71 zákona č. 111/1998 Sb., o vysokých školách
a o změně a doplnění dalších zákonů
(zákon o vysokých školách)

Vás jmenuji
s účinností od 7. prosince 2018

DOCENTEM

pro obor

APLIKOVANÁ MATEMATIKA



V Brně dne 7. prosince 2018
Č.j. 68/90230/2018

prof. RNDr. Ing. Petr Štěpánek, CSc.
rektor

doc. RNDr. Michal Novák, Ph.D.

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Education and scientific titles

doc. 2018; Brno University of Technology, Faculty of Mechanical Engineering

Applied Mathematics

RNDr. 2007; Faculty of Sciences, Masaryk University

Upper Secondary School Teacher Training in Mathematics

Ph.D. 2004; Faculty of Sciences, Masaryk University

General Problems of Mathematics

MSc. 1998; Faculty of Sciences, Masaryk University

Upper Secondary School Teacher Training: Mathematics – English

Employment history

since 2000: Brno University of Technology

Faculty of Electrical Engineering and Communication, Department of Mathematics

Head of the Department (starting from April 2021)

Deputy Department Head (2010 – 2021)

Professional specialization

theory of hypercompositional structures (algebraic hyper-structures) and their applications; MSC 2020 codes 20N20 (Hypergroups), 16Y20 (Hyperrings), 06Fxx (Ordered structures), 68Q70 (Algebraic theory of languages and automata), 20M35 (Semigroups in automata theory, linguistics, etc.)

Scientific activities in the last 5 years related to the topic of the project proposal

a) Research results:

- 10 papers in journals with impact factor (Web of Science), additional 1 accepted for publication in 2021
- 2 conference papers indexed by Web of Science, additional 3 conference papers indexed by SCOPUS
- 1 book chapter indexed by Web of Science
- 2 papers in peer-review journals and 1 conference paper unindexed by Web of Science or SCOPUS

b) Projects:

- Team member of FEKT-S-17-4225: *Dynamics of systems with the focus also on their algebraic and topological structure* (2017-2019)

c) Membership in editorial boards:

- Italian Journal of Pure and Applied Mathematics (SCOPUS), since December 2020
- Journal of Intelligent and Fuzzy Systems (IF WoS: Q3), since January 2019

d) Reviews for scientific journals:

- *Indexed by Web of Science or SCOPUS*: European Journal of Combinatorics; Soft Computing; Open Mathematics; International Journal of Biomathematics; Turkish Journal of

Mathematics; Symmetry; Kragujevac Journal of Mathematics; Discrete Mathematics, Algorithms and Applications; Applied Mathematics and Information Sciences

- Other: Mathematical Reviews; Southeast Asia Bulletin of Mathematics; Cogent Mathematics and Statistics; Mathematical Sciences Letters;

e) Research stays and mobilities:

- University of Nova Gorica (Slovenia): 10 stays
- Eastern Macedonia and Thrace University of Technology (TEI Kavala) (Greece): 2 stays

Bibliography

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2. Krehlik, Š., Novák, M., Modified product of automata as a better tool for description of real-life systems(2020) AIP Conference Proceedings, 2293, art. no. 340003, .
3. Novák, M., Křehlík, Š., Staněk, D., n-ary Cartesian composition of automata(2020) Soft Computing, 24 (3), pp. 1837-1849.
4. Bordbar, H., Novák, M., Cristea, I., A note on the support of a hypermodule(2020) Journal of Algebra and its Applications, 19 (1), art. no. 2050019.
5. Cristea, I., Kocijan, J., Novák, M., Introduction to dependence relations and their links to algebraic hyperstructures(2019) Mathematics, 7 (10), art. no. 885.
6. Chvalina, J., Novák, M., Křehlík, Š., Hyperstructure generalizations of quasi-automata induced by modelling functions and signal processing(2019) AIP Conference Proceedings, 2116, art. no. 310006.
7. Novák, M., Křehlík, Š., Ovaliadis, K., Elements of hyperstructure theory in UWSN design and data aggregation(2019) Symmetry, 11 (6), art. no. 734.
8. Novák, M., Cristea, I., Composition in EL-hyperstructures(2019) Hacettepe Journal of Mathematics and Statistics, 48 (1), pp. 45-58.
9. Novák, M., Ordering in the Algebraic Hyperstructure Theory: Some Examples with a Potential for Applications in Social Sciences(2019) Studies in Systems, Decision and Control, 179, pp. 535-551.
10. Novák, M., Křehlík, Š., Cristea, I., Cyclicity in EL-hypergroups(2018) Symmetry, 10 (11), art. no. 611.
11. Novák, M., Křehlík, Š., EL-hyperstructures revisited(2018) Soft Computing, 22 (21), pp. 7269-7280.
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13. Bordbar, H., Novák, M., Cristea, I., Jun, Y.B., Properties of reduced meet ideals in lower BCK-semilattices(2018) 17th Conference on Applied Mathematics, APLIMAT 2018 - Proceedings, 2018-February, pp. 97-109.

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17. Chvalina, J., Křehlík, Š., Novák, M., Cartesian composition and the problem of generalizing the MAC condition to quasi-multiautomata (2016) Analele Stiintifice ale Universitatii Ovidius Constanta, Seria Matematica, 24 (3), pp. 79-100.
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vzdělávání, Sborník 29. konference o matematice na VŠTEZ. Zlín: FAI UTB Zlín, 2006. p. 17 (p.)ISBN: 80-7318-450- 8.

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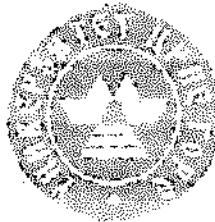
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На основу члана 75 stav 2 Zakona o visokom obrazovanju (Sl.list RCG br. 60/03) i člana 18 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore, na sjednici održanoj 25.03.2010. godine, donio je

ОДЛУКУ О ИЗБОРУ У ЗВАНЈЕ

Dr SVJETLANA TERZIĆ bira se u akademsko званје **редовни професор** Univerziteta Crne Gore za predmete: Uvod u geometriju i Algebarska topologija na osnovnom studijskom programu Matematika i Uvod u diferencijalnu geometriju na osnovnom studijskom programu Matematika i računarske nauke na **Pриродно-математичком факултету**.

РЕКТОР

Мирјановић Предраг
Prof.dr Predrag Miranović

Biografija: Svetlana Terzić

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Pol: ženski

Oblasti istraživanja: Algebarska topologija, Diferencijalna geometrija

Akademski obrazovanje:

- Diplomirani matematičar – Univerzitet Crne Gore, 09. 1993., srednja ocjena 9,96 od ukupno 10.
- Magistar matematike – Univerzitet u Beogradu, 06. 1996., srednja ocjena 10 od ukupno 10
Magistarska teza “Karakteristične klase hiperkompleksnih vektorskih raslojenja”, mentor Prof. Novica Blažić
- Doktor matematike – Moskovski državni univerzitet “M. V. Lomonosov”, 02. 1996 – 07. 1998,
srednja ocjena 5 od ukupno 5
Doktorska disertacija “Realne kohomoologije i karakteristične klase uopštenih simetričnih prostora ”, menotor Prof. Yuri P. Solovyov, 06. 1998.
- Postdoktorska pozicija, 08. 2000- 08. 2002, Ludwig Maximilians University, Minhen, Njemačka

Akademска звјанја:

- 1993 -2000 – saradnik u naставi, Univerzitet Crne Gore
- 2000-2005 - Docent, Univerzitet Crne Gore
- 2005-2010 – Vanredni profesor, Univerzitet Crne Gore
- 2010- Redovni profesor, Univerzitet Crne Gore
- 2011 –Vanredni član Crnogorske akademije nauka i umjetnosti
- 2018 – Redovni član Crnogorske akademije nauka i umjetnosti

Odabrana predavanja po pozivu:

1. Contemporary Geometry and Related Topics, Belgrade, Serbia and Montenegro, May 2002;
2. Kolmogorov and Contemporary Mathematics, Moscow, Russia, Jun 2003;
3. Mathematical, Theoretical and Phenomenological Challenges Beyond Standard Models, Vrnjačka Banja, Serbia and Montenegro, September 2003;
4. Algebraic models for topological spaces and fibrations, Tbilisi, Georgia, September 2004
5. XI congress of mathematicians of Serbia and Montenegro, Petrovac, Serbia and Montenegro , September 2004, plenary talk;
6. Topology, analysis and applications to mathematical physics, Moscow, Russia, February 2005;
7. Contemporary Geometry and Related Topics, Belgrade, Serbia and Montenegro, July 2005;
8. Toric Topology, Osaka, Japan, May, 2006;
9. Differential Equations and Topology, in commemoration of the 100th anniversary of L.S. Pontryagin, Moscow, Russia, Jun 2008;
10. New horizons in toric topology, Manchester, UK, July 2008;
11. Geometry, Dynamics, Integrable systems, Belgrade, Serbia, September 2008;
12. Multidisciplinarnost i jedinstvo savremene nauke, University of East Sarajevo, Pale, May 2009;
13. Geometry, topology and algebra, dedicated to 120th anniversary of Boris Delone, Steklov Mathematical Institute, Russian Academy of Science, Moscow, Russia, August , 2010;
14. Geometry, Dynamics, Integrable Systems, Belgrade, Serbia, September, 2010;
15. Toric topology and automorphic functions, Khabarovsk, Far eastern branch of Russian academy of science, September, 2011
16. International topological conference “Alexandroff readings, Moscow state university “M. V. Lomonosov”, May 2012, plenary talk
17. The second mathematical conference of the Republic of Srpska, Trebinje, Jun, 2012, plenary talk
18. Geometric structure on manifolds and their applications, Castle Raufschholzhausen, Marburg, July, 2012.
19. XVII geometrical seminar, Zlatibor, Serbia, August, 2012.

21. International conference "Algebraic topology and Abelian function" in honor of Victor Buchstaber on occasion of his 70th birthday, Moscow, June, 2013.
22. Geometry and analysis of metric structures, Sobolov institute of mathematics, Russian Academy of Sciences, Novosibirsk, December, 2013.
23. Topology of torus actions and its applications to geometry, Satelite conference of ICM, Daejeon, Korea, August, 2014.
24. International conference "Torus actions in geometry, topology and applications, Skolkovo, Moscow, February, 2015.
25. The fifth mathematical conference of the Republic of Srpska, Trebinje, Jun 2015.
26. International Chinese-Russian conference "Torus actions: topology, geometry and number theory, Beijing, China, October, 2015.
27. Aspects of Homotopy Theory, Southampton, UK, December 2015.
28. XIX Geometrical Seminar, Zlatibor, Serbia, September 2016
29. Mini conference celebrating of 30 years of CGTA seminar, Belgrade, Serbia, September 2016, plenary talk
30. The Princeton-Rider Workshop on the Homotopy Theory of Polyhedral products, Princeton and Rider University, Princeton, USA, May-June, 2017.
31. Symposium on mathematics and it applications, Belgrade, Serbia, November 2017.
32. International conference "Algebraic topology, Combinatorics and Mathematical Physics" in honor of Victor Buchstaber on occasion of his 75th birthday, Moscow, May, 2018.

33. International conference "Modern algebra and Analysis and their Applications, Academy of Sciences and Arts of Bosnia and Herzegovina, Sarajevo, September, 2018.

34. Susret matematičara Srbije i Crne Gore, Budva, Oktobar, 2019.

35. Toric topology 2019 in Okayama, Okayamo, Japan, Novembar, 2019.

36. Deseti simpozijum Matematika i primene, Beograd, Decembar, 2019

37. Workshop on Torus actions in Topology, Fields Institute, Toronto, Kanada, May, 2020, via zoom.
38. Workshop on toric topology, geometry and related subjects, Moscow, November , 2020, via zoom

Predavanja na seminarima:

- September 2002., Erwin-Schroedinger institute, Vienna, Austria, talk in the framework of the program Aspects of foliation theory;
- April 2005, SANU, Belgrade, talk at the na Mathematical Colloquium SANU;
- Jun 2006, Osaka City University, Japan, talk at the Topology seminar;
- Jul 2006, University of Aberdeen, UK, talk at the Topology seminar
- January 2007, University of Oxford, UK, talk at the Topology seminar, mini course for phd topology students on the rational minimal model theory;
- February 2007, University of Manchester, UK, talk at the Topology seminar;
- November 2007, Mathematical Institute SANU, Belgrade, talk at the Geometry seminar;
- April 2009, MFO (Oberwolfach), Germany , talk at the "Workshop on homotopy theory of function spaces and related topics";
- September 2009, Faculty of Mechanics and Mathematics, MSU "M. V. Lomonosov", Moscow, Russia, talk at the seminar for Geometry, topology and mathematical physics, chaired by V. M. Buchstaber and S. P. Novikova , talk at the Chair seminar of A. T. Fomenko;
- Mart 2010, Laboratori J. A. Dieudonne; Universite de Nica Sophia Antipolis, France, talk at the seminar for Algebra, topology and geometry;
- Jun 2010, International School for Advanced Studies SISSA, Trieste, Italy, talk at the seminar for Geometry and Physics chaired by B. A. Dubrovin;
- December 2011, SANU, talk at the seminar Mathematical methods of mechanics.
- September 2013, University of Southampton, talk at Topology seminar
- December 2013, SANU, talk at the seminar Mathematical methods of mechanics
- October 2016, Faculty of Mechanics and Mathematics, MSU "M. V. Lomonosov", Moscow, Russia, talk at the seminar for Geometry, topology and mathematical physics, chaired by V. M. Buchstaber and S. P. Novikov
- May 2017, University of Southampton, UK; talk at Topology seminar
- December 2018, University of Southampton, UK, talk at Topology seminar

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- November 2019, talk at One day topology seminar in Osaka, Osaka, Japan
 - Oktobar 2020, University of Southampton, talk at Topology seminar, via zoom
 - Novembar 2020, Princeton University, talk at International Polyhedral Product seminar, via zoom

Odabране nagrade i grantovi:

1. Nagrada 19. decembar za najboljeg studenta u generaciji 1991.
2. Plaketa Univerziteta Crne Gore za najboljeg diplomiranog studenta generacije, 1993.
3. Nagrada Crnogorske akademije nauka i umjetnosti za naučna dostignuća, 2003.
4. Grant Evropskog udruženja matematičara za učešće na IV Evropskom kongresu matematičara, Štokholm, Švedska, 2004.
5. WUS-Austria 2-nedjeljna posjeta Jelene Grbić Podgorici, Crna Gora, April, 2007.
6. Oxford Colleges hospitality scheme, 1-mjesečna posjeta Univerzitetu u Oxford-u, Januar, 2007.

7. Grant Evropskog udruženja matematičara za učešće na V Evropskom kongresu matematičara, Amsterdam, Holandija, 2008.
8. Grant of the Medjunarodne matematičke unije za učešće na Svjetskom kongresu matematičara, Hyderabad, Indija, 2010.
9. Bilateralni projekat sa Univerzitetom u Ljubljani, Slovenija, 2012-2013.
10. Glavni istraživač na projektu Ministarstva nauke Crne Gore, 2012-2015.
11. Glavni istraživač na medjunarodnom projektu instituta SISSA, Trst, 2008-2010.
12. Spoljni istraživač na projektu 174020 Ministarstva nauke Srbije, 2011-2015.
13. Istraživački grant London Mathematical Society sa Jelenom Grbić, 2-nedjeljna posjeta Univerzitetu u Southampton-u, 2013.
14. Grant za istraživanje u parovima sa Jelenom Grbić, 3-nedjeljna posjeta Matematičkom institutu Oberwolfach, 2014.
15. Grant Medjunarodne matematičke unije za učešće na Svjetskom matematičkom kongresu Rio de Janeiro, Brazil, 2018.

Neke naučno istraživačke posjete:

Mehaniko-matematički fakultet, Moskovski državni univerzitet, Matematički institut Steklova, Ruska akademija nauka – 1999, 2003, 2005, 2006, 2008, 2009, 2012, 2013, 2015, 2016, 2018; SISSA, Trst 2010; Matematički Fakultet, Ljubljana, 2012, 2013; Matematički fakultet, Univerzitet u Southampton-u, 2013, 2015, 2017; Univerzitet u Aberdeen-u, 2006; Univerzitet u Mančester-u, 2007. Fildsov Institut za matematiku, Toronto, Kanada, 2020.

Nastava i mentorstvo:

Predavala kurseve na različitim nivoima studija na Prirodno-matematičkom fakultetu Univerziteta Crne Gore: Uvod u geometriju, Uvod u diferencijanu geometriju, Algebarska topologija, Diferencijalna geomatrija na mnogostrukostima, Geometrija, Napredna algebra.

Mentor za preko 20 specijalističkih radova i 4 magistraske teze, komentor doktorske disertacije na Matematičkom fakultetu, Univerzitet Nica Sophia Antipolis, član-komisija za odbranu doktorskih disertacija na Univerzitetu u Beogradu, Univerzitetu u Istočnom Sarajevu, Univerzitetu u Southampton-u, Univerzitetu Crne Gore.

Ostalo:

- Urednik:
 1. Sarajevo Journal of Mathematics, izdaje Akademija nauka i umjetnosti Bosne i Hercegovine
 2. Matematički Vesnik, izdaje Društvo matematičara Srbije
- Recenzent za časopise : Publication de l'Institut Mathématique, Contemporary Mathematics, Proceedings of the Steklov Institute of Mathematics, Annali di Matematica Pura ed Applicata, Mathematica Slovaca, Mathematische Zeitschrift, Sbornik Mathematics, Algebraic and Geometric Topology, Homology, Homotopy and Applications, Moroccan Journal of Pure and Applied Analysis.
- Prodekan za medjunarodnu sardanju na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, 2004 – 2007.

Publication list for Svjetlana Terzić

1. Svjetlana Terzić, *Real cohomology and Pontryagin characteristic classes of generalised symmetric spaces*, (Russian) Vsesojuzni Institut Nauchnoj i Tehnicheskoy Informacii, VINITI, V-1034, Moscow, 1998, 1-94.
2. Svjetlana Terzić, *Generalised symmetric spaces and their topology*, (Russian) Mathematica Montisnigri 11 (1999), 139-150.
3. Svjetlana Terzić, *Characteristic classes of hypercomplex vector bundles*, Montenegrin Academy of Sciences and Arts, Proceeding of the Section of Natural Sciences, 13 (2000)
4. Svjetlana Terzić, *Cohomology with real coefficients of generalized symmetric spaces*, (Russian) Fundamental'naya i Prikladnaya Matematika, Vol. 7, (2001), no. 1, 131-157.
5. Svjetlana Terzić, *Pontryagin classes of generalized symmetric spaces*, (Russian) Matematicheskie Zametki, Vol. 69, (2001), no.4, 613-621; English transl. in Mathematical Notes, Vol. 69, (2001), no. 4, 559-566.
6. D. Kotschick and S. Terzić, *On formality of generalised symmetric spaces*, Mathematical Proceedings of Cambridge Philosophical Society, 134 (2003), 491-505.
7. S. Terzić, *Rational homotopy groups of generalised symmetric spaces*, Mathematische Zeitschrift, 243 (2003), 491-523.
8. S. Terzić, *On rational topology of four manifolds*, Proceeding of the Workshop Contemporary Geometry and Related Topics, World Scientific 2004, 375-389.
9. Svjetlana Terzić, *Rational topology of gauge groups and of spaces of connections*, Compositio Mathematicae, 141 (2005), no.1, 262-270.
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Na osnovu člana 97. Zakona o Univerzitetu ("Sl.list RCG", br. 27/92 i 6/94) i člana 94. Statuta Univerziteta Crne Gore, Naučno-nastavno vijeće Univerziteta Crne Gore, na sjednici održanoj 25.12.2003.godine, donijelo je

ОДЛУКУ О ИЗБОРУ У ЗВАНЈЕ

Dr BILJANA ZEKOVIĆ bira se u zvanje redovnog profesora Univerziteta Crne Gore za predmete: Algebra i Matematika na nematičnim fakultetima na Prirodno-matematičkom fakultetu u Podgorici.



РЕКТОР,

Prof.dr Ljubiša Stanković

- 1 Algebra sa primenama (2005-2007)
2. Neke algebarske strukture (2008-2011)
3. Algebarske strukture sa primenama (2012-2014)

Aktivni istraživač na projektu **Algebra i logika sa primenama (2000-2005)**

Biografija

Rođena sam 7.2.1955. u Gajdobri, opština Bač. Palanka. U Prištini sam završila osmogodišnju školu i gimnaziju prirodno-matematičkog smera 1974. godine oba puta kao nosilac Vukove diplome. Diplomirala sam na PMF-u 1978. godine. Postdiplomske studije sam upisala u jesen te iste godine na PMF-u u Prištini i Skoplju (integrisane postdiplomske studije) i za dve godine položila sve ispite predviđene planom i programom postdiplomskih studija. Magistarski rad sam odbranila na PMF-u u Prištini 1.10.1982. pod nazivom "S-disjunktivni elementi semigrupa sa posebnim osvrtom na neke klase semigrupa."

Školsku godinu 1987/88. provela sam na usavršavanju na MGU u Moskvi, gde sam sa naučnim rukovodiocem V.A.Artamonovim radila na nekim problemima iz oblasti Univerzalne algebre, što je rezultiralo izradom doktorske disertacije, koju sam pod nazivom "Prilog teoriji n-polugrupa i (2,n)-prstena", odbranila 2. marta 1990. godine na PMF-u u Skoplju pod rukovodstvom prof. dr G.Čupone. Kasnije, u okviru saradnje Moskovskog državnog univerziteta-MGU i Univerziteta Crne Gore, bila sam više puta na studijskom boravku na MGU im. M.V. Lomonosava.

Od 1.10.1978. do 1.10.1982. godine radila sam na PMF-u u Prištini, gde sam u zvanju asistenta držala vežbe iz Algebре i Matematičkih predmeta na Tehničkim fakultetima kao i na Ekonomskom fakultetu. Od 1.10. 1982. godine radim na PMF-u u Podgorici, gde sam držala vežbe iz Algebре i Matematičkih predmeta na Tehničkim fakultetima u zvanju asistenta sve do izbora u zvanje docenta 27.2.1991. Od tada sam na Odseku za fiziku držala predavanja i vežbe iz Diferencijalnog i integralnog računa I i II i Linearne algebре i analitičke geometrije, predavanja iz Matematike na Arhitekturi, Matematike I i II na Metalurškom fakultetu, predavanja i vežbe iz Algebре na Odseku za matematiku i računarske nauke (smer C), predavanja iz Agebre II (smer A i B) i Algebре III (smerA).

U zvanje vanrednog profesora izabrana sam 5.10.1998, a u zvanje redovnog profesora 25.12.2003. godine.

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Radovi prezentovani na naučnim konferencijama i pripremljeni za štampu

19. B. Zeković, **(Ко) Инварианты n-арных (ко) алгебр и тензорные произведения n- арных биалгебр, pripremljen za štampu, a prezentovan na The Ninth International Seminar "Mathematical Models & Modeling in Laser-Plasma Processes and Advanced science technologies", Petrovac, Montenegro, 28. maj - 4. jun, 2011.**

20. B. Zekovich, **Some properties of the algebra $R(C)$ and Morita equivalence**, The Thirteenth International Seminar "Mathematical Models & Modeling in Laser - Plasma Processes and Advanced Science Technologies", Petrovac, Montenegro, 2015.
21. B. Zekovich, **The properties of tensor-products of the irreducible moduls over n-bialgebras**, Fourteenth International Seminar "Mathematical Models & Modeling in Laser-Plasma Processes and Advanced Science Technologies", Moscow, Russia, 1-12 July, 2016.
22. B. Zekovich, **Initial consideration about tensor product of irreducible modules over n-bialgebras**, Mathematical and Informational Technologies, MIT-2016
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24. B. Zekovich, V. A. Artamonov, **Semisimple n-ary bialgebras and one-dimensional module**, The Fifteenth and the Sixteenth International Seminar "Mathematical Models & Modeling in Laser-Plasma Processes and Advanced Science Technologies", Petrovac, Montenegro, 2017.
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26. B. Zekovich, V. A. Artamonov, **Pierce decomposition of semisimple n-ary bialgebras**, The Seventeenth International Seminar "Mathematical Models & Modeling in Laser-Plasma Processes and Advanced Science Technologies", Budva, Montenegro, 2018.
27. B. Zekovich, **On tensor-products of irreducible moduls over n-bialgebras**, Međunarodna algebarska konferencija na mehaničko-matematičkom fakultetu, posvećena 110-godišnjici rođenja prof. A.G. Kuroša
28. B. Zekovich, V. A. Artamonov, **Antipode in n-ary bialgebra**, The Eighteenth International Seminar "Mathematical Models & Modeling in Laser-Plasma Processes and Advanced Science Technologies", Petrovac, Montenegro, 2019.
29. B. Zekovich, V. A. Artamonov, **Coinvariant elements and connection with antipode**, pripremljen za štampu
30. B. Zekovich, **Some consideration about semisimple decomposition of tensor products of irreducible modules over n-bialgebras**, Moskva, Rusija, 2019.

Spisak objavljenih knjiga

1. B. Zeković, V.A.Artamonov, **Zbirka rešenih zadataka iz Algebri (prvi deo)**, PMF, Podgorica 2003.
2. B. Zeković, V.A.Artamonov, **Zbirka rešenih zadataka iz Algebri (drugi deo)**, (recenziran)

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Na osnovu člana 72 stav 2 Zakona o visokom obrazovanju („Službeni list Crne Gore“ br. 44/14, 47/15,40/16,42/17,71/17 55/18 i 3/19) i člana 32 stav 1 tačka 9 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore, na sjednici održanoj 19. aprila 2019.godine, donio je

O D L U K U O IZBORU U ZVANJE

Dr SANJA JANČIĆ-RAŠOVIĆ bira se u akademsko zvanje redovni profesor Univerziteta Crne Gore za **oblast Matematika** (Algebra 1) **na Prirodno-matematičkom fakultetu Univerziteta Crne Gore i** (Matematika 3 i Matematika sa informatikom) **na nematičnim fakultetima**, na neodređeno vrijeme.

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- Sc Matematika, Univerzitet Crne Gore, Prirodno-matematički fakultet GPA: 10.00/10.00, 2002.
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- 2008-2013 Docent na Univerzitetu Crne Gore
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III STUDIJSKI BORAVCI:

- Jun 2007- Department of Algebra, Johannes Kepler University, Linz, Austrija
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IV STRUCNI RAD:

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 - (2005-2007) "Algebra sa primjenama", projekat je podržan od strane Ministarstva prosvjete i nauke Crne Gore.
 - (2008-2011) "Neke algebarske strukture", projekat je podržan od Ministarstva prosvjete i nauke Crne Gore.
 - (2006-2007) "Teaching methods in Mathematics" project, No. 204/2006, WUS-Austria Project.

- (2012-2014) "Algebarske strukture sa primjenama" projekat je podržan od Ministarstva nauke Crne Gore.
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3. Učešće u radu komisija Ispitnog centra Crne Gore (2012-...)

V UNIVERZITETSKI UDŽBENICI:

- Radoje Sćepanović, Sanja Jančić Rašović: *Matematika 3 za studente građevinskog i mašinskog fakulteta*, Univerzitet Crne Gore 2012.godine, ISBN 978-86-7664-103-1
- Radoje Sćepanović, Sanja Jančić Rašović: *Matematika za studente arhitekture*, Univerzitet Crne Gore 2009.godine, ISBN 978-86-7664-067-6
- S. Duborija, M. Mosurović, G. Šuković, S. Jančić: *Diferencijalni i integralni račun : Zbirka ispitnih zadataka*, Univerzitet Crne Gore, 1999.godine, ISBN 86-81039-43-1.

VI NAUCNI RADOVI

1. Sanja Jančić Rašović, Irina Cristea, *Hypernear-rings with a defect of distributivity*, *Filomat, Niš, Volume 32(4), (2018), pp.1133-1149, ISSN:0354-5180. (SCI Expanded)*
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4. Sanja Jančić-Rašović, Vučić Dašić, Some new classes of (m,n)-hyperrings, *Filomat, Niš, Srbija, ISSN:0354-5180, Vol.26,Issue 3, 585-596 (2012) (SCI Expanded)*.
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6. Sanja Jančić Rašović, Vučić Dašić, On generalization of division near-rings, *Italian Journal of Pure and Applied Mathematics, Udine, Italy, No-40, pp.1-8, ISSN: 2239-0227,(SCOPUS)*
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9. Irina Cristea, Sanja Jančić Rašović and Sohrab Ostadhadi-Dehkordi, How to define tensor products of hypermodules, *International Conference on Numerical Analysis and Applied Mathematics* 2017 (ICNAAM 2017), 25-30 September 2017, Thessaloniki, Greece, AIP Conference Proceedings 1978, 34003 (2018), doi: 10.1063/1.5043946, pp.34003-1-34003-4., ISBN:978-0-7354-1690-1, Publisher American Institute of Physics.
10. Sanja Jančić Rašović, On hyperrings associated with L-fuzzy relations, *Mathematica Montisnigri*, Vol XXIV (2012), p.137-149, ISSN:0354-2238, (bazapodataka : Zentralblatt Math).
11. Sanja Jančić Rašović, About the hyperring of polynomials – *Italian Journal of Pure and Applied Mathematics* – N. 21-(2007) ,223-234, Udine, Italy, ISSN:2239-0227, (SCOPUS).
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